Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

■ : explanation
∵ is 'because'
∴ is 'therefore'

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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P 6 5 7 9 2 A 0 1 3 2

Pearson

1. The curve C has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(4)

The point $P(4, 3)$ lies on C.

(b) Find the equation of the normal to C at the point P. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

a) ① rewrite equation in easier form for integration

$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 = \frac{1}{3}x^2 + \frac{4}{\sqrt{x}} + \frac{8}{3} \times \frac{1}{x} - 5$

$= \frac{1}{3}x^2 + \frac{4}{\sqrt{x}} + \left(\frac{8}{3}\right)\left(\frac{1}{x}\right) - 5 = \frac{1}{3}x^2 + \frac{4}{x^{1/2}}$ ① $+ \left(\frac{8}{3}\right)\left(\frac{1}{x}\right) - 5$

① indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$= \frac{1}{3}x^2 + \frac{4}{x^{1/2}} + \left(\frac{8}{3}\right)\left(\frac{1}{x^1}\right) - 5 = \frac{1}{3}x^2 + 4x^{-1/2} + \frac{8}{3}x^{-1} - 5$ ②

② indicies rule $\frac{x}{a^b} = xa^{-b}$

$x^0 = 1$

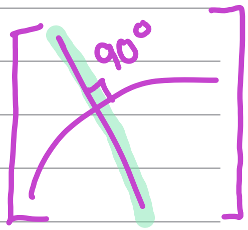
② differentiate $y = \frac{1}{3}x^2 + 4x^{-1/2} + \frac{8}{3}x^{-1} - 5$

$dy/dx = 2\left(\frac{1}{3}x^{2-1}\right) + (-1/2)\left(4x^{-1/2-1}\right) + (-1)\left(8/3\, x^{-1-1}\right) - 0\left(5x^{0-1}\right)$

$= \frac{2}{3}x - \frac{4}{2}x^{-3/2} - \frac{8}{3}x^{-2}$

$\therefore dy/dx = \frac{2}{3}x - 2x^{-3/2} - \frac{8}{3}x^{-2}$

b) normal is Perpendicular to Curve



$\therefore$ we find gradient of normal (m_n)

using Perpendicular gradient rule $M_{normal} \times M_{curve} = -1$

Question 1 continued

① to find gradient of Curve, Substitute x-value of P into dy/dx (the gradient function)

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{2}{3}(4) - 2(4)^{-3/2} - \frac{8}{3}(4)^{-2} = \frac{9}{4}$$

from part (a)

② find gradient of normal using formula.

$$M_n \times M_c = -1$$

$$\div \frac{9}{4} \quad M_n \times 9/4 = -1 \quad \div \frac{9}{4}$$

$$M_n = -4/9$$

③ find equation of normal using line passing through (a, b) and gradient M

equation: $(y-b) = M(x-a)$

$a = 4$

$b = 3$

$M = -4/9$

$$(y-3) = -4/9\,(x-4)$$

④ write in form $ax+by+c=0$

$$\times 9 \quad y-3 = -4/9\,(x-4) \quad \times 9$$

$$9y-27 = -4(x-4)$$

$$-16 \quad 9y-27 = -4x+16 \quad -16$$

$$9y-43 = -4x$$

$$+4x \quad 4x+9y-43 = 0 \quad +4x$$

$\therefore$ $4x+9y-43=0$ $\quad a=4 \quad b=9 \quad c=-43$

Q1

(Total 8 marks)

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P 6 5 7 9 2 A 0 3 3 2

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = ax^3 + (6a + 8)x^2 - a^2x$$

where a is a positive constant.

Given $f(-1) = 32$

(a) (i) show that the only possible value for a is 3

(ii) Using $a = 3$ solve the equation

$$f(x) = 0$$

(5)

(b) Hence find all real solutions of

(i) $3y + 26y^{\frac{2}{3}} - 9y^{\frac{1}{3}} = 0$

(ii) $3(9^{3z}) + 26(9^{2z}) - 9(9^{z}) = 0$

(5)

a)i) $f(-1) = a(-1)^3 + (6a+8)(-1)^2 - a^2(-1) = 32$

$-a + (6a+8) + a^2 = 32$

$a^2 + 5a + 8 = 32$

$a^2 + 5a - 24 = 0$

factorise : $(a+8)(a-3) = 0$

Solve : $a+8=0 \rightarrow a=-8$

$a-3=0 \rightarrow a=3$

a cannot be -8 $\because$ a is a positive constant $\therefore$ only possible value of a is 3.

ii) $f(x) = 3x^3 + (6(3)+8)x^2 - (3)^2x = 0$

$3x^3 + 26x^2 - 9x = 0$

factorise : $3x^3 + 26x^2 - 9x = 0$

$x(3x^2 + 26x - 9) = 0$

$x(3x-1)(x+9) = 0$

Solve : $x = 0$

$x+9=0 \rightarrow x=-9$

$3x-1=0 \rightarrow 3x=1 \rightarrow x=\frac{1}{3}$

$\therefore x = -9, 0, \frac{1}{3}$

P 6 5 7 9 2 A 0 4 3 2

Question 2 continued

b)i) let $y^{1/3} = x$ → $y^{2/3} = (y^{1/3})^2 = x^2$

$y^1 = (y^{1/3})^3 = x^3$

$\therefore\ 3x^3 + 26x^2 - 9x = 0$

$x = -9, 0, 1/3$ ← from part (a)

$x = y^{\frac{1}{3}} = -9$ cube, $y = (-9)^3$ cube, $y = -729$

$x = y^{\frac{1}{3}} = 0$ cube, $y = 0$ cube

$x = y^{1/3} = 1/3$ cube, $y = (1/3)^3$ cube, $y = \frac{1}{27}$

$\therefore\ y = -729,\ 0,\ \frac{1}{27}$

ii) let $9^z = x$ → $9^{2z} = (9^z)^2 = x^2$

$9^{3z} = (9^z)^3 = x^3$

$\therefore\ 3x^3 + 26x^2 - 9x = 0$

$x = -9, 0, 1/3$ ← from part (a)

$x = 9^z \neq -9$ not possible

$x = 9^z \neq 0$ not possible

$x = 9^z = 1/3$

$9^z = 1/3$

can solve with this if you have done Pure 2 Maths

$\left(\begin{array}{c} a^x = n \\ \log_a n = x \end{array} \right)$

if you know logarithms (Pure 2 Maths) $\log_9 -9$ & $\log_9 0$ ARE UNDEFINED

$\because$ can't do log of zero or negative number

Pure 1 only : $9^z = 1/3$

$9^z = 3^{-1}$ indicies rule $\frac{x}{a^b} = xa^{-b}$

$3^{2z} = 3^{-1}$

$2z = -1$

$\therefore\ z = -1/2$

P 6 5 7 9 2 A 0 5 3 2

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Question 2 continued

P 6 5 7 9 2 A 0 6 3 2

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Question 2 continued

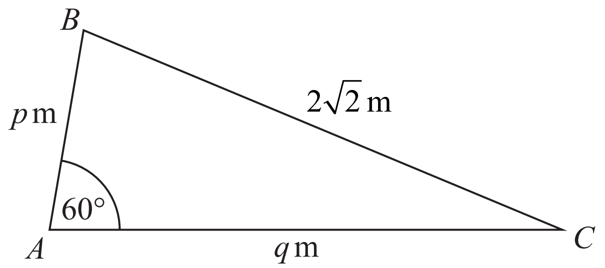
Q2

(Total 10 marks)

P 6 5 7 9 2 A 0 7 3 2

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.



Not drawn to scale

Figure 1

Figure 1 shows the plan view of a flower bed.
The flowerbed is in the shape of a triangle ABC with

- $AB = p$ metres
- $AC = q$ metres
- $BC = 2\sqrt{2}$ metres
- angle $BAC = 60°$ ← UNITS!!

(a) Show that

$$p^2 + q^2 - pq = 8$$

(2)

Given that side AC is 2 metres longer than side AB, use algebra to find

(b) (i) the exact value of p,

(ii) the exact value of q.

(5)

Pure Mathematics P1

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

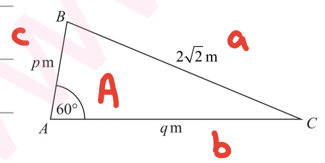
$a^2 = b^2 + c^2 - 2bc\cos A$

Using the answers to part (b),

(c) calculate the exact area of the flower bed.

(2)

a) Use Cosine rule



$$(2\sqrt{2})^2 = q^2 + p^2 - 2(q)(p)\cos 60°$$
$$8 = q^2 + p^2 - 2pq\left(\tfrac{1}{2}\right)$$
$$\therefore\ p^2 + q^2 - pq = 8$$

b) $AC = AB + 2$

$q = p + 2$

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Question 3 continued

i) substitute $q = p+2$ into equation from part (a)

$p^2 + (p+2)^2 - p(p+2) = 8$

$p^2 + (p^2+4p+4) - (p^2+2p) = 8$

$p^2 + p^2 + 4p + 4 - p^2 - 2p = 8$

$p^2 + 2p + 4 = 8$ -8

$1p^2 + 2p - 4 = 0$

Solve using quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = 2$

$c = -4$

$\dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)} = \dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm \sqrt{4 \times 5}}{2}$

$= \dfrac{-2 \pm \sqrt{4} \times \sqrt{5}}{2} = \dfrac{-2 \pm 2\sqrt{5}}{2} = \dfrac{2(-1 \pm \sqrt{5})}{2}$

$= -1 \pm \sqrt{5}$

$P > 0$ $\because$ P is length of side AB & length CANNOT be negative.

$\therefore P \neq -1 - \sqrt{5}$

$\therefore P = -1 + \sqrt{5}$

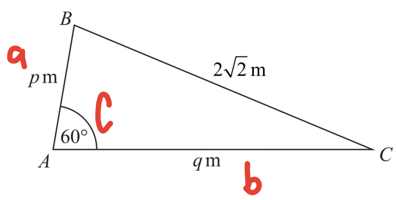
ii) find q by substituting P

$q = p + 2$

$q = (-1 + \sqrt{5}) + 2 = 1 + \sqrt{5}$

$\therefore q = 1 + \sqrt{5}$

c) Area of a triangle : $A = \frac{1}{2} ab \sin C$



$A = \frac{1}{2} \times p \times q \times \sin 60$

$= \frac{1}{2} \times (-1+\sqrt{5})(1+\sqrt{5}) \times \frac{\sqrt{3}}{2} = \frac{1}{2} \times 4 \times \frac{\sqrt{3}}{2} = 2 \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

$\therefore$ Area $= \sqrt{3}\ m^2$

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DO NOT WRITE IN THIS AREA

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Turn over

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Question 3 continued

Leave blank

Question 3 continued

(Total 9 marks)

Q3

P 6 5 7 9 2 A 0 1 1 3 2

Turn over

4. Find

$$\int \frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}}\,dx$$

writing each term in simplest form.

(6)

① Write in easier form for integration

$$\frac{(3\sqrt{x}+2)(x-5)}{4\sqrt{x}} = \frac{(3x^{1/2}+2)(x-5)}{4x^{1/2}}$$ ①

① indicies rule $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$3x^{1/2} \times x^1$ ②

$$\frac{(3x^{1/2}+2)(x-5)}{4x^{1/2}} \underset{\text{brackets}}{\overset{\text{expand}}{=}} \frac{(3x^{1/2+1} - 15x^{1/2} + 2x - 10)}{4x^{1/2}}$$

② indicies rule $a^b \times a^c = a^{b+c}$

$$\frac{3x^{3/2} - 15x^{1/2} + 2x - 10}{4x^{1/2}} = \frac{1}{4}\left(\frac{3x^{3/2} - 15x^{1/2} + 2x - 10}{x^{1/2}}\right)$$

$x^0 = 1$

$$= \frac{1}{4}\left(3x^{3/2-1/2} - 15x^{1/2-1/2} + 2x^{1-1/2} - 10x^{0-1/2}\right)$$ ③

③ indicies rule $\frac{a^b}{a^c} = a^{b-c}$

$$\frac{1}{4}\left(3x - 15 + 2x^{1/2} - 10x^{-1/2}\right) = \frac{3}{4}x - \frac{15}{4} + \frac{2}{4}x^{1/2} - \frac{10}{4}x^{-1/2}$$

$$\therefore \frac{3}{4}x - \frac{15}{4} + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}$$

② Integrate

$$\int \frac{3}{4}x^1 - \frac{15}{4}x^0 + \frac{1}{2}x^{1/2} - \frac{5}{2}x^{-1/2}\,dx$$

$$= \left[\left(\frac{3/4}{1+1}x^{1+1}\right) + \left(\frac{-15/4}{0+1}x^{0+1}\right) + \left(\frac{1/2}{1/2+1}x^{1/2+1}\right) + \left(\frac{-5/2}{-1/2+1}x^{-1/2+1}\right)\right]$$

$$= \frac{3}{8}x^2 - \frac{15}{4}x^1 + \frac{1}{3}x^{3/2} - 5x^{1/2} + C$$

↳ DON'T FORGET or will lose marks

P 6 5 7 9 2 A 0 1 2 3 2

Question 4 continued

$\therefore \frac{3}{8}x^2 - \frac{15}{4}x + \frac{1}{3}x^{\frac{3}{2}} - 5x^{\frac{1}{2}} + C$

Q4

(Total 6 marks)

Turn over

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5.

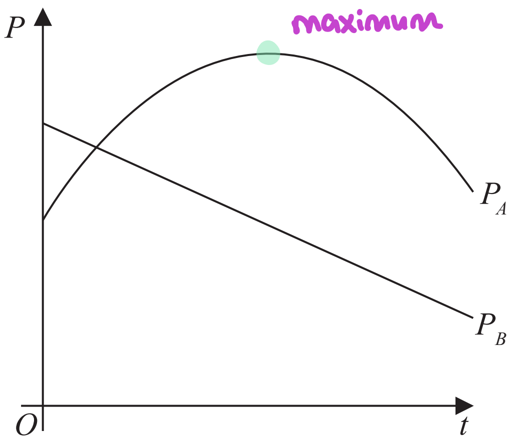


Figure 2

The share value of two companies, company A and company B, has been monitored over a 15-year period.

↙ units

The share value P_A of **company** $\boldsymbol{A}$, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

The share value P_B of **company** $\boldsymbol{B}$, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \qquad t \geqslant 0$$

where t is the number of years after monitoring began.

Figure 2 shows a graph of both models.

Use the equations of one or both models to answer parts (a) to (d).

(a) Find the difference between the share value of **company** $\boldsymbol{A}$ and the share value of **company** $\boldsymbol{B}$ at the point monitoring began.

↳ initial time $\therefore t = 0$

(2)

(b) State the maximum share value of **company** $\boldsymbol{A}$ during the 15-year period.

(1)

(c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company** $\boldsymbol{A}$ was greater than the share value of **company** $\boldsymbol{B}$.

(4)

(d) Explain why the model for **company** $\boldsymbol{A}$ should not be used to predict its share value when $t = 20$

(1)

a) Monitoring began $t = 0$.

$P_B - P_A = (-1.6(0) + 44.2) - (53 - 0.4(0-8)^2)$

$= 44.2 - 27.4 = £16.8$ million

Question 5 continued

b) $P_A = 53 - 0.4(t-8)^2$ when $t = -8$, $P_A = 53$

$\because$ Subtracting zero from 53 provides largest value for P_A.

$\therefore P_A = £53$ million

c) $P_A > P_B$ $\longrightarrow$ $53 - 0.4(t-8)^2 > -1.6t + 44.2$

$53 - 0.4(t^2 - 16t + 64) > -1.6t + 44.2$

$53 - 0.4t^2 + 6.4t - 25.6 > -1.6t + 44.2$

$-0.4t^2 + 6.4t + 27.4 > -1.6t + 44.2$

$+1.6t$ $\quad -0.4t^2 + 8t + 27.4 > 44.2 \quad +1.6t$

-44.2 $\quad -0.4t^2 + 8t - 16.8 > 0 \quad -44.2$

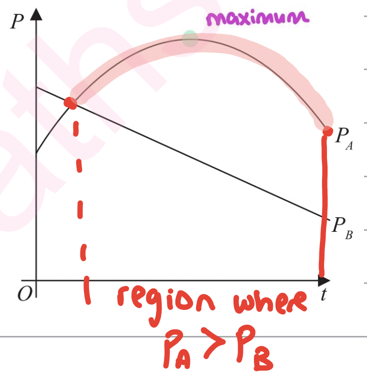
$\div -0.4$ $\quad t^2 - 20t + 42 < 0 \quad \div -0.4$

$\because$ divided by negative number

$1t^2 - 20t + 42 = 0$

Solve to find critical values

we want to find the 2 t values shown.



Solve using quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = -20$

$c = 42$

$\dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(42)}}{2(1)} = \dfrac{20 \pm \sqrt{232}}{2}$

$= \dfrac{20 \pm \sqrt{4 \times 58}}{2} = \dfrac{20 \pm \sqrt{4} \times \sqrt{58}}{2} = \dfrac{20 \pm 2\sqrt{58}}{2}$

$\therefore t = 10 \pm \sqrt{58}$

BUT $10 + \sqrt{58} > 15$ & graph is only for 15 year period $\therefore t \le 15$

$\therefore 10 - \sqrt{58} < t \le 10 + \sqrt{58}$

NOT $\le$ $\because$ P_A intersects with P_B at this point

Turn over

Question 5 continued

$10 - \sqrt{58} = 2.3842\ldots \approx 2.38$ years

$\therefore 2.38 < t \leq 15$

d) Model is stated to be only for 15 years and $20 > 15$.

OR

$P_A = 53 - 0.4(20 - 8)^2 = -4.6$

when $t = 20$, Share value would be negative, but $P \geqslant 0$

P 6 5 7 9 2 A 0 1 6 3 2

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Question 5 continued

(Total 8 marks)

Q5

P 6 5 7 9 2 A 0 1 7 3 2

6. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$

Given that

- C passes through the point $P(8, 2)$
- $\mathrm{f}'(x) = \dfrac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $\mathrm{f}(x)$.

(5)

a) tangent means gradient of tangent is same as gradient of equation

① to find gradient of tangent, substitute x-value of P into $f'(x)$ (the gradient function)

$$f'(8) = \frac{32}{3(8)^2} + 3 - 2\left(\sqrt[3]{(8)}\right) = -\frac{5}{6}$$

② find equation of tangent using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = 8$
$b = 2$
$m = -\frac{5}{6}$

$$(y - 2) = -\frac{5}{6}(x - 8)$$

③ Write in the form $y = mx + c$

$$y - 2 = -\frac{5}{6}x + \frac{20}{3} \quad +2$$
$$y = -\tfrac{5}{6}x + \tfrac{26}{3}$$

$$\therefore \boxed{y = -\frac{5}{6}x + \frac{26}{3}}$$

Question 6 continued

b) $f'(x) \underset{\text{differentiate}}{\overset{\text{integrate}}{\rightleftarrows}} f(x)$

① Write in form easier for integration

$$f'(x) = \frac{32}{3x^2} + 3 - 2(\sqrt[3]{x}) = \frac{32}{3}x^{-2} + 3 - 2x^{\frac{1}{3}}$$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$ ② indicies rule: $\frac{x}{a^b} = xa^{-b}$

② integrate

$\because x^0 = 1$

$$f(x) = \int f'(x)\,dx = \int \frac{32}{3}x^{-2} + 3x^0 - 2x^{1/3}\,dx$$

$$= \left[\left(\frac{32/3}{-2+1}x^{-2+1}\right) + \left(\frac{3}{0+1}x^{0+1}\right) + \left(\frac{-2}{\frac{1}{3}+1}x^{1/3+1}\right)\right]$$

$$= -\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{4/3} + C$$

③ find $+C$ using $P(8, 2)$

$$f(8) = 2 = -\frac{32}{3}(8)^{-1} + 3(8) - \frac{3}{2}(8)^{4/3} + C$$

$$2 = -\frac{4}{3} + C \quad \left(+\frac{4}{3}\right)$$

$$\frac{10}{3} = C$$

$$\therefore f(x) = -\frac{32}{3}x^{-1} + 3x - \frac{3}{2}x^{\frac{4}{3}} + \frac{10}{3}$$

Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

7.

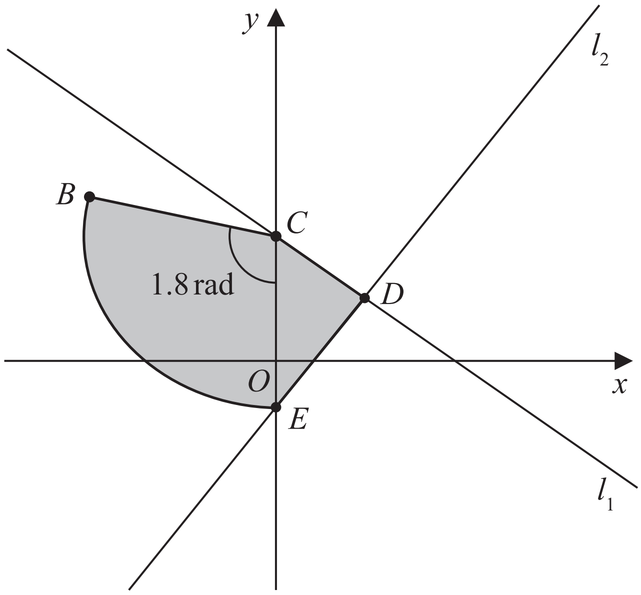


Figure 3

The line l_1 has equation $4y + 3x = 48$

The line l_1 cuts the y-axis at the point C, as shown in Figure 3.

(a) State the y coordinate of C.

(1)

The point $D(8, 6)$ lies on l_1

The line l_2 passes through D and is perpendicular to l_1

The line l_2 cuts the y-axis at the point E as shown in Figure 3.

(b) Show that the y coordinate of E is $-\dfrac{14}{3}$

(3)

A sector BCE of a circle with centre C is also shown in Figure 3.

Given that angle BCE is 1.8 radians,

(c) find the length of arc BE.

(3)

The region $CBED$, shown shaded in Figure 3, consists of the sector BCE joined to the triangle CDE.

(d) Calculate the exact area of the region $CBED$.

(3)

a) y-coordinate when $x = 0$

$4y + 3(0) = 48 \longrightarrow 4y = 48$

$\div 4 \quad y = 12 \quad \div 4$

$\therefore$ y-coordinate is 12

Question 7 continued

b) $l_1 \perp l_2$ (perpendicular) $\therefore$ l_2 is normal to l_1

We need to find equation of l_2 to find point E.

① To find gradient, use perpendicular gradient rule

$$M_{normal} \times M_{line} = -1$$

first find gradient of l_1 M_{line} by rearranging equation into $y = mx + c$

$$l_1 : 4y + 3x = 48$$

$$y = \boxed{-\tfrac{3}{4}}x + 12$$

$$m_1 = -\tfrac{3}{4}$$

find l_2 gradient :

$$\div -\tfrac{3}{4} \quad M_n \times -\tfrac{3}{4} = -1 \quad \div -\tfrac{3}{4}$$

$$M_n = \tfrac{4}{3}$$

② Find equation of tangent using line passing through (a , b) and gradient M

equation : $(y - b) = M(x - a)$ use point D(8, 6) $\because$ intersection

$a = 8$

$b = 6$

$M = \tfrac{4}{3}$

$$(y - 6) = \tfrac{4}{3}(x - 8)$$

③ find y-intercept when $x = 0$

$$y - 6 = \tfrac{4}{3}(0 - 8)$$

$$y - 6 = \tfrac{4}{3}(-8)$$

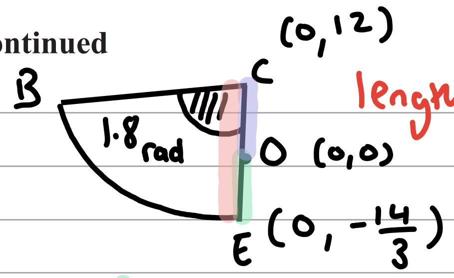
$$+6 \quad y - 6 = -\tfrac{32}{3} \quad +6$$

$$y = -\tfrac{14}{3}$$

$\therefore$ y-Coordinate is $-\tfrac{14}{3}$

Question 7 continued

c)



length of arc : $S = r\theta$

radius $= 12 + \frac{14}{3} = \frac{50}{3}$

$S = \left(\frac{50}{3}\right)(1.8) = 30$ units

$\therefore$ $S = 30$ units

d) Area CBED = Area Sector BCE + Area triangle CDE

① Sector BCE

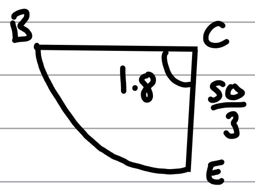
$\downarrow$

250 units^2

Area of sector : $A = \frac{1}{2}r^2\theta$

$A = \left(\frac{1}{2}\right)\left(\frac{50}{3}\right)^2(1.8) = \underline{\underline{250}}$

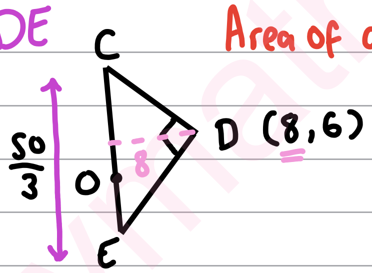
from part (c)



② Triangle CDE

$\downarrow$

$\frac{200}{3}$ units^2



Area of a triangle : $A = \frac{1}{2}bh$

$A = \frac{1}{2} \times \frac{50}{3} \times 8 = \frac{200}{3}$

③ Area CBED = Area Sector BCE + Area triangle CDE

$= 250 + \frac{200}{3}$

$= \frac{950}{3}$

$\therefore$ Area$_{CBED} = \frac{950}{3}$ units^2

P 6 5 7 9 2 A 0 2 4 3 2

Leave blank

Question 7 continued

Q7

(Total 10 marks)

P 6 5 7 9 2 A 0 2 5 3 2

8. The curve C_1 has equation

$$y = 3x^2 + 6x + 9$$

(a) Write $3x^2 + 6x + 9$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The point P is the minimum point of C_1

(b) Deduce the coordinates of P.

(1)

A different curve C_2 has equation

$$y = Ax^3 + Bx^2 + Cx + D$$

where A, B, C and D are constants.

Given that C_2

- passes through P
- intersects the x-axis at -4, -2 and 3

(c) find, making your method clear, the values of A, B, C and D.

(5)

a) Completing the square:

if $y = x^2 + bx + c$

$$y = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$$

$y = 3x^2 + 6x + 9$

$y = 3(x^2 + 2x + 3)$

$$y = 3\left(\left(x + \frac{2}{2}\right)^2 + 3 - \left(\frac{2}{2}\right)^2\right)$$

$y = 3\left((x+1)^2 + 2\right)$

$\therefore\ y = 3(x+1)^2 + 6$ $\quad a=3\ \ b=1\ \ c=6$

b) To find minimum point: $y = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

inverse is x-coordinate $\quad$ y-coordinate

Question 8 continued

Explanation: when $y = c-(b/2)^2$ $\quad c-(b/2)^2 = (x+b/2)^2 + c-(b/2)^2$

$0 = (x+b/2)^2$

$\therefore x = -b/2$

when $x = -b/2$ $\quad y = (-b/2 + b/2)^2 + c-(b/2)^2$

$\therefore y = c - (b/2)^2$

$y = 3(x+1)^2 + 6$

$-x \qquad y$

$\therefore$ Minimum $(-1, 6)$

c) C_2 intersects x-axis at $-4, -2, 3$.

$\therefore$ when $y=0$, $x = -4, -2, 3$

This can be represented by: $y = \alpha(x+4)(x+2)(x-3)$
where 'α' is a constant to be found.

C_2 passes $P(-1, 6)$ from part (b)

Substitute into equation & find α

$6 = \alpha(-1+4)(-1+2)(-1-3)$

$\div -12 \big(\quad 6 = -12\alpha \quad \big) \div -12$

$\frac{6}{-12} = \alpha$

$\therefore \alpha = -\frac{1}{2}$

Expand brackets to find A, B, C, D

$y = -\frac{1}{2}(x+4)(x+2)(x-3)$

$y = -\frac{1}{2}(x+4)(x^2 - x - 6)$

$y = -\frac{1}{2}(x^3 + 3x^2 - 10x - 24)$

$\therefore y = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 5x + 12$

$A = -\frac{1}{2} \quad B = -\frac{3}{2} \quad C = 5 \quad D = 12$

P 6 5 7 9 2 A 0 2 7 3 2

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

P 6 5 7 9 2 A 0 2 9 3 2

9.

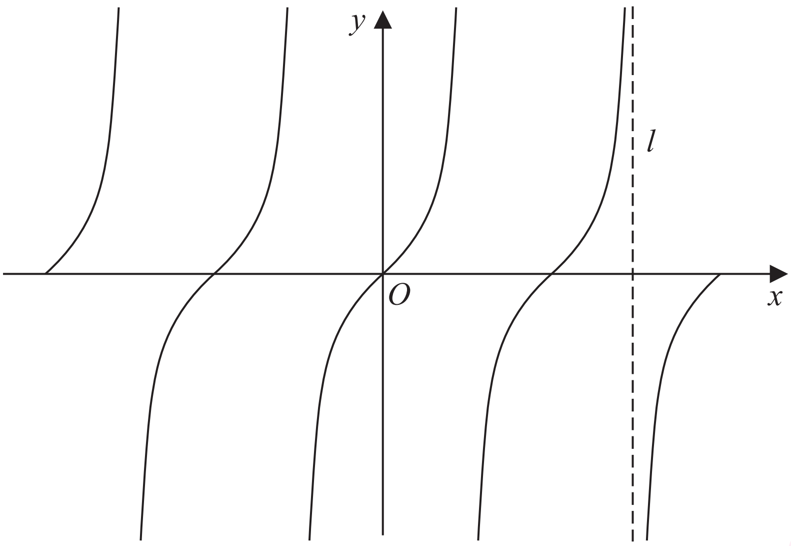


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \qquad -2\pi \leqslant x \leqslant 2\pi$$

indicates that we are dealing with radians.

The line l, shown in Figure 4, is an asymptote to $y = \tan x$

(a) State an equation for l.

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \qquad -2\pi \leqslant x \leqslant 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region $-2\pi \leqslant x \leqslant 2\pi$

(4)

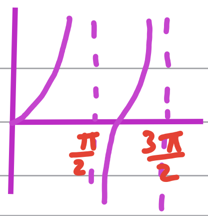
(c) State the number of solutions of the equation $\tan x = \dfrac{1}{x} + 1$ in the region

(i) $0 \leqslant x \leqslant 40\pi$

(ii) $-10\pi \leqslant x \leqslant \dfrac{5}{2}\pi$

(2)

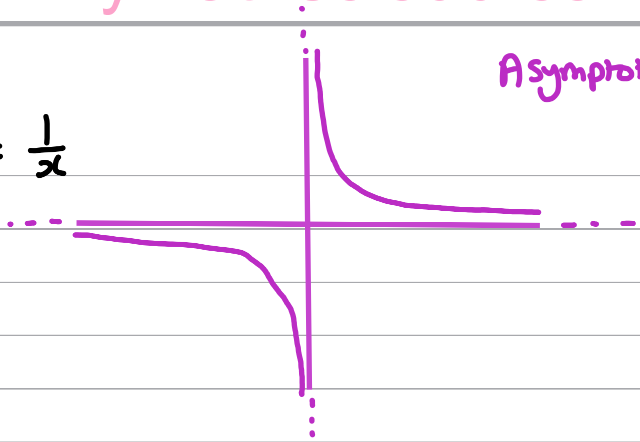
9a) $y = \tan x$



$\therefore\ l : \boxed{x = \dfrac{3\pi}{2}}$

Question 9 continued

b)i) working : $y = \frac{1}{x}$

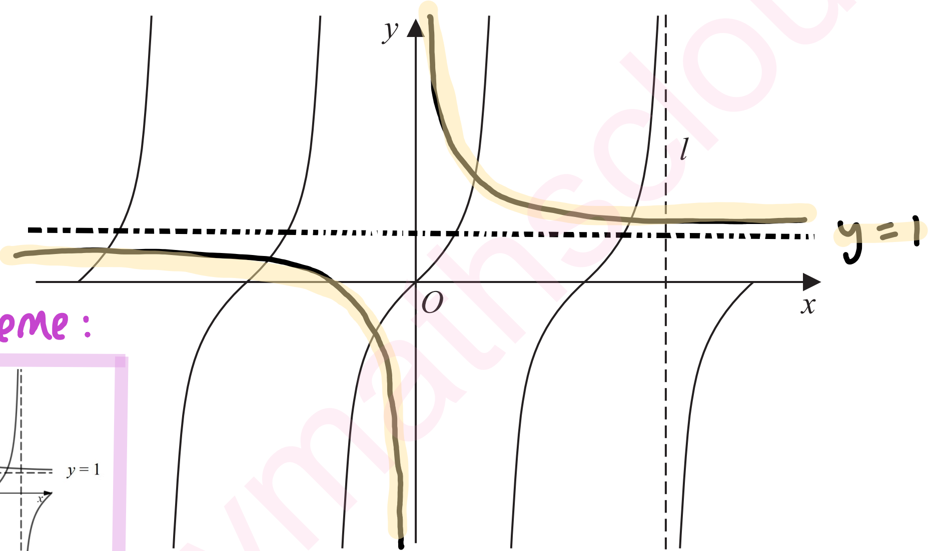


$y = \frac{1}{x} + 1$

is translation of graph $y = \frac{1}{x}$ by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, 1 unit up.

Asymptotes : $x = 0$

$y = 0 + 1 = 1$



Mark scheme :

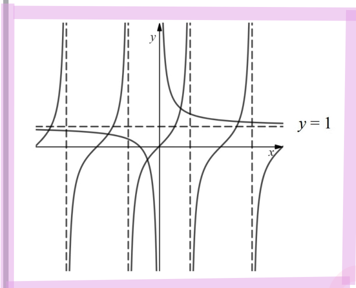
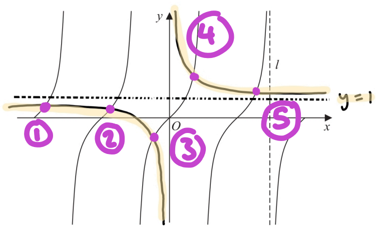


Diagram 1

ii) 5 Solutions $\because$ number of solutions is number of points of intersection between the graphs.



c)i) number of solutions in region $0 \leq x \leq 40\pi$

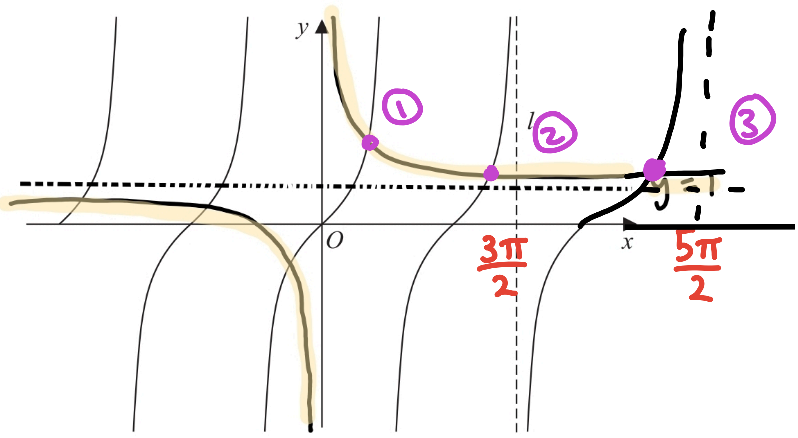
in $0 \leq x \leq 2\pi$ there are 2 intersections, so 2 solutions

$2\pi \xrightarrow{\times 20} 40\pi$ $\therefore$ 2 solutions $\xrightarrow{\times 20}$ 40 solutions

$\therefore$ 40 solutions

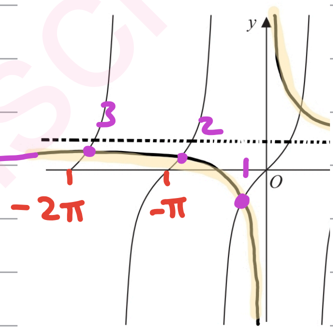
Question 9 continued

ii) number of solutions in region $-10\pi \le x \le \frac{5}{2}\pi$



for $0 < x \le \frac{5\pi}{2}$, 3 intersections $\therefore$ 3 solutions

As for $-10\pi \le x < 0$:



in region $-2\pi \le x \le \pi$
2 intersections.

$-2\pi \xrightarrow{\times 5} -10\pi$ $\quad$ $2 \xrightarrow{\times 5} 10$

10 intersections in region $-10\pi \le x \le \pi$

$\therefore$ 11 intersections in region $-10\pi \le x \le 0$ $\therefore$ 11 solutions

$11 + 3 = 14$

$\therefore$ In region $-10\pi \le x \le \frac{5\pi}{2}$ there are 14 solutions

Q9

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END

P 6 5 7 9 2 A 0 3 2 3 2